



Tuts from Additive Combinatorics

Def.  $A(f) = \{(i_1, \dots, i_k) \mid x_1^{i_1} \dots x_k^{i_k} \text{ is a monomial of } f\}$ , its convex hull  $N(A)$  is called Newton polytope of  $f$

Prop.  $A(f^m) = mA$

Prop. (Khanziki q2).  $\exists N$ . when  $m > N$ .  $\exists f_k(m)$  s.t.  $|mA(f)| = f_k(m)$  and  $LC(f_k(m)) = U(N)$ .

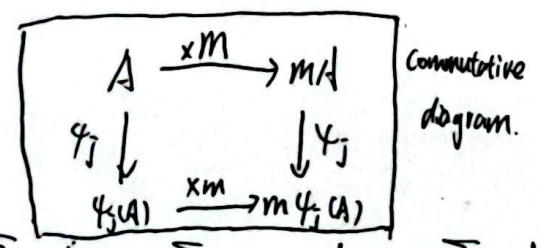
$$J_e = \{x_1^{i_1} \dots x_k^{i_k} \mid (i_1, \dots, i_k) \in mA \text{ \& } (i_1, \dots, i_k) \in (m-1)A + \alpha\}$$

$\downarrow$   
LM(f)

$$\dim(L) = \left| \bigcup_{t=0}^m J_e \right| = |J_0| = |mA(f)| \xrightarrow{\text{Khanziki q2}} U(N(f)) m^k + o(m^k)$$

$$\det(L) = x_1^{P_1(m)} \dots x_k^{P_k(m)} M^{P_F(m)} = x_1^{\int_{mA(f)} x_i dV_{m^{k+1}}} \dots x_k^{\int_{N(f)} x_k dV_{m^{k+1}}} M^{\frac{k}{k+1} U(N(f)) m^{k+1} + o(m^{k+1})}$$

$$P_j: \sum_{(i_1, \dots, i_k) \in mA} i_j \xrightarrow{?} |mA_j|$$



$$\psi_j: \mathbb{Z}^k \rightarrow \mathbb{Z}^{k+1}$$

$$(i_1, \dots, i_k) \mapsto (i_1, \dots, i_k, 0), \dots, (i_1, \dots, i_k, i_j)$$

$$\sum_{(i_1, \dots, i_k) \in mA} i_j = \sum_{(i_1, \dots, i_k, i_{k+1}) \in \psi_j(mA)} 1 = \sum_{(i_1, \dots, i_k) \in \psi_j(A)} 1$$

$$P_F: \text{Construct } \tilde{A} \in \mathbb{Z}^{k+1}, \sum_{t=0}^m (m-t) |J_e| |J_{e+1}| \stackrel{?}{=} |m\tilde{A}|$$

$$\sum_{t=0}^m (m-t) |J_e| |J_{e+1}| = \sum_{t=0}^m (m-t) (|J_e| - |J_{e+1}|) \xrightarrow{\text{Abel's summation formula}} m|J_0| - \sum_{t=0}^m |J_e|$$

$\downarrow$   
 $m|mA|$        $???$

$$\tilde{A} = (A, 1) \cup (\alpha, 0). \sum_{t=0}^m ((m-t)A + t\alpha) = |m\tilde{A}| \stackrel{?}{=} \sum_{t=0}^m |mA|, U(N(\tilde{A})) = \frac{1}{k+1} U(N(A))$$

Generalized to a system of polynomial system

$$F = \{f_1, \dots, f_n\} \quad A(F) = \bigcup_{j=1}^n A(f_j) \quad S_e = \{x_1^{i_1} \dots x_k^{i_k} \mid (i_1, \dots, i_k) \in (m-1)A(F) + U \sum_{j=1}^n \alpha_j \}$$

$$g_{(i_1, \dots, i_k)}(x_1, \dots, x_k) = \frac{x_1^{i_1} \dots x_k^{i_k}}{\prod_{j=1}^n LM(f_j)^{i_j}} \prod_{j=1}^n f_j^{i_j} M^{m-l}$$

$$\text{Add. } t\text{-shift} \quad J_e = \bigcup_{0 \leq j \leq e} \{x_1^{i_1} \dots x_k^{i_k} \mid \sum_{l=1}^k i_l = e\} \quad \dim(L) = |mA + tE|$$

$$\text{CSURF: } X < M^{\frac{5t-3n+7}{45}} \approx M^{0.25812}$$